91054

B.Sc.–I 1st Semester (Hons.) Examination,

November–2014

MATHS

Paper–BHM–112

Calculus

Time allowed : 3 hours] *[Maximum marks : 60*

Note : *Attempt **five** questions in all, selecting **one** from each section. **Question No. 9 is compulsory.***

Section–I

1. (a) By definition, prove that

$$f(x) = \begin{cases} x^2 \cos\frac{1}{x}, & x \neq 0 \\ 0 & x = 0 \end{cases}$$ is continuous at $x = 0$.

(b) Find the nth derivative of $e^x \sin^2 x \sin 2x$.

2. (a) If $y = e^{m \cos^{-1} x}$, prove that

$$(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} - (n^2 + m^2) y_n = 0$$

(b) Obtain the values of sin 31° upto four places of decimals.

Section–II

3. (a) Find all the asymptotes of the curve

$$x^3 + 2x^2y + xy^2 - x^2 - xy + 2 = 0$$

(b) Find the asymptotes of the curve $r \cos \theta = a \sin^2\theta$.

4. (a) In the cycloid $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$ prove that $P = 4a \cos \frac{\theta}{2}$.

(b) Show that at the point $(1, -1)$, there is a cusp on the curve $x^3 + xy^2 + y^3 - 4x^2 + y^2 + 4x + y - 1 = 0$

Section–III

5. (a) Trace the curve $r = a(1 + \sec \theta)$

(b) Find a reduction formula for $\int \frac{x^m}{(\log x)^n} dx$.

6. (a) Connect :

$\int x^m (a + b x^n)^p dx$ with $\int x^m (a + b x^n)^{p-1} dx$.

(b) Show that the length of the loop of the curve $3ay^2 = x(x - a)^2$ is $4a/\sqrt{3}$

Section–IV

7. (a) Show that the area of the loop of the curve $ay^2 = (x - a)(x - 5a)^2$ is $\frac{256}{15} a^2$.

(b) Find the area of the loop of the curve $x = a(1-t^2)$ $y = at(1-t^2)$.

8. (a) Find the volume of the solid formed by the revolution about the x-axis of the curve $y^2(a+x) = x^2(3a-x)$.

(b) Find the surface area of the solid generated by revolving one arc of the curve $x = a(\theta - \sin\theta)$; $y = a(1-\cos\theta)$ about x-axis.

Section–V

9. (a) Evaluate $\lim_{x\to 0^+} \frac{x}{|x|}$

(b) Find nth derivative of $\frac{1}{ax+b}$

(c) Define node and cusp.

(d) Evaluate $\int_0^{\pi/2} \sin^6\theta \, d\theta$

(e) Find the area of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(f) What is axes of revolution ?